

Masses and decay widths of radially excited Bottom mesons

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Abstract

Inspired from the experimental information coming from LHC [2,3] and Babar [4] for radially higher excited charmed mesons, we predict the masses and decays of the $n=2$ S-wave and P-wave bottom mesons using the effective lagrangian approach. Using heavy quark effective theory approach, non-perturbative parameters ($\bar{\Lambda}$, λ_1 and λ_2) are fitted using the available experimental and theoretical informations on charm masses. Using heavy quark symmetry and the values of these fitted parameters, the masses of radially excited even and odd parity bottom mesons with and without strangeness are predicted. These predicted masses led in constraining the decay widths of these 12 states, and also shed light on the unknown values of the higher hadronic coupling constants \tilde{g}_{SH}^2 and \tilde{g}_{TH}^2 . Studying the properties like masses, decays of 2S and 2P states and some hadronic couplings would help forthcoming experiments to look into these states in future.

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1 Introduction

In the past decade, many new discoveries have filled the spectroscopy of charmed and bottom mesons. Recent discovery in 2015, by LHCb collaboration on bottom states [1] diverts theorists interest towards the study of bottom sector. Observing the bottom spectroscopy, it is realized that unlike the success in charm sector, the experimental information for bottom sector is missing. Till now experimental information available on bottom mesons is only for $n=1$ ground and excited state only which is as shown in Table 1 [5].

New resonances $B(5970)^+$ and $B(5970)^0$ are found in the mass distribution of $B^0\pi^+$ and $B^+\pi^-$ respectively. Masses and decay widths of these resonances as predicted by CDF collaboration [6] in 2013 are as

$$\begin{aligned} M(B(5970)^+) &= 5961 \pm 13 \text{ MeV} \\ \Gamma(B(5970)^+) &= 60 \pm 6 \text{ MeV} \\ M(B(5970)^0) &= 5977 \pm 13 \text{ MeV} \\ \Gamma(B(5970)^0) &= 70 \pm 40 \text{ MeV} \end{aligned}$$

$J^P(n^{2s+1}L_J)$	Experimentally known masses(MeV)			
	Bottom meson		Charm Meson	
	Non-Strange	Strange	Non-Strange	Strange
$0^-(1^1S_0)$	5279.61/5279.37	5366.81	1869.61/1864.84	1968.30
$1^-(1^3S_1)$	5324.83	5415.4	2010.27/2006.97	2112.10
$0^-(2^1S_0)$	5840* [1]	-	2580* [3, 4]	-
$1^-(2^3S_1)$	5960* [6]	-	2680* [3, 4]	2709* [7]
$0^+(1^3P_0)$	-	-	2403/2318	2317.7
$1^+(1^1P_1)$	-	-	2427	2459.5
$1^+(1^3P_1)$	5724.9/5726.8	5828.4	2421.4	2535.11
$2^+(1^3P_2)$	5739/5735.9	5839.87	2464.3/2462.6	2571.9
$0^+(2^3P_0)$	-	-	3000* [3, 4]	-
$1^+(2^1P_1)$	-	-	3000* [3, 4]	3040* [8]
$1^+(2^3P_1)$	-	-	3000* [3, 4]	3040* [8]
$2^+(2^3P_2)$	-	-	3000* [3, 4]/3214* [2]	-
$1^-(1^3D_1)$	-	-	2760* [3, 4]	2860* [9]
$2^-(1^1D_2)$	-	-	2740* [3, 4]	-
$2^-(1^3D_2)$	5840* [1]	-	2760* [3, 4]	-
$3^-(1^3D_3)$	5960* [6]	-	2740* [3, 4]	2860* [10]

Table 1: Experimentally available Bottom and Charm Meson Masses. First value in Columns 2 and Column 4 is for $Q\bar{d}$ and second value is for $Q\bar{u}$ where Q is the heavy quark i.e. Q=b/c. Values without * are taken from PDG [5]. States with * have been experimentally observed, but still their accurate J^P is yet to be assigned.

Since the resonances decay in $B\pi$ final states, they are expected to have natural spin-parity states which is still to be confirmed. Many theorists suggest this state to be $B^*(2^3S_1)$ [11, 12]. Li-Ye Xio using chiral quark model suggests it to be 1^3D_3 bottom state [13]. And the $B_J(5840)$ state reported by LHCb [1] is suggested to fill the $B(2^1S_0)$ state with $\Gamma(B_J(5840)) = 175.9$ MeV and $M(B_J(5840)) = 5857$ MeV [14]. Many theoretical predictions on the masses and the decay widths of bottom and charm mesons have been made [15–19]. These predictions are based on various models like constituent quark model [15], pseudo-scalar emission model [16], chiral quark model [17], 3P_0 model [18], heavy quark effective theory [19] etc. But these different theoretical predictions are not uniform as these different models uses different parameters to predict the masses of various states. As in the non relativistic quark model, Hamiltonian is introduced which includes various input parameters like r (the separation between the two quarks), σ , α_s , μ , γ_E etc. As all these are theoretical parameters, so its different input values generates different mass spectra. And in the framework of HQET, most of the prediction on masses and the strong decay widths is available for $n=1$ states. Information on masses and decays about the higher states $n=2$ for strange and non strange sector is not known clearly due to the presence of extra couplings of the higher orders. Observing the Table 1, We are motivated to compute the higher states masses of the spectrum so that we can predict their decay widths and then can put some constrain on higher hadronic couplings. In this paper, we made the prediction for masses for $n=2$ strange and non- strange bottom and charm mesons for H, S and T fields using the HQET as our model.

HQET provides heavy-light meson mass prediction in terms of few unknown QCD non-perturbative parameters at a given order of $\frac{1}{m_Q}$ [20]. These parameters $\bar{\Lambda}$, λ_1 and λ_2 represents the operators of the HQET lagrangian at the first order $\frac{1}{m_Q}$ expansion. The information on decays and mass Out of these parameters, λ_1 gives kinetic energy of the heavy quark and λ_2 gives the chromomagnetic interaction for the heavy quark. Previous study on these parameters has provided some range to their value [21,22] , but this data is least available for $n=2$ or higher states. More information about the data for $n=2$ or higher states is required to take its value in confidence. Recently theorists have also predicted the masses of these $n=2$ states by using mixing concept. According to this, states with same spin and parity can mix for e.g. 3P_1 and 1P_1 state can mix as these states have same spin and parity. Spectroscopy of bottom mesons attained by this concept, has been shown using models like non-relativistic quark model [14], constituent quark model [23] etc. As it can be seen from the literature that the values predicted by different models are very much deviating from one another. Hoping that our calculations provide some insight to our framework, we proceed as follow: In section 2, a brief review about the theory used i.e. "Heavy Quark Effective Theory" is given. This description includes the information about the importance of these non-perturbative parameters. This is followed by the section 3, in which fit these parameters to predict the strange and non-strange bottom meson masses. These predicted masses are verified by calculating their strong decay widths in terms of some hadronic coupling constants followed by the conclusion in the last section.

2 Framework

In the framework of the heavy quark effective theory, hadrons containing single heavy quark are analyzed. This theory is an effective QCD theory for N_f heavy quarks Q with mass $m_Q \gg \Lambda_{QCD}$, with heavy quark Q 's four velocity fixed [24]. In this theory, spin and parity of the heavy quark decouples from the light degrees of freedom quarks as they interact through the exchange of soft gluons only. Heavy mesons are classified in doublets in relation to the total conserved angular momentum *i.e.* $s_l = s_{\bar{q}} + l$, where $s_{\bar{q}}$ and l are the spin and orbital angular momentum of the light anti-quark respectively. For $l = 0$ (S-wave) the doublet is represented by (P, P^*) with $J_{s_l}^P = (0^-, 1^-)_{\frac{1}{2}}$, which for $l = 1$ (P-wave), there are two doublets represented by (P_0^*, P_1') and (P_1, P_2^*) with $J_{s_l}^P = (0^+, 1^+)_{\frac{1}{2}}$ and $(1^+, 2^+)_{\frac{3}{2}}$ respectively. Two doublets of $l = 2$ (D-wave) are represented by (P_1^*, P_2) and (P_2', P_3^*) belonging to $J_{s_l}^P = (1^-, 2^-)_{\frac{3}{2}}$ and $(2^-, 3^-)_{\frac{5}{2}}$ respectively. These doublets are described by the effective super-field H_a, S_a, T_a, X_a, Y_a [32], where the field H_a describe the (P, P^*) doublet *i.e.* S-wave, S_a and T_a fields represents the P-wave doublets $(0^+, 1^+)_{\frac{1}{2}}$ and $(1^+, 2^+)_{\frac{3}{2}}$ respectively. D-wave doublets are represented by the X_a and Y_a fields. For the radial excitation of these states with radial quantum number $n=2$, these states are replaced by \tilde{P}, \tilde{P}^* and so on. Thus the properties of the hadrons are invariant under $SU(2N_f)$ transformations, *i.e.* heavy quark spin and flavor symmetries providing a clear picture in the study of the heavy quark physics. These symmetries are exploited to study the charm and bottom meson spectra and are shown by the QCD lagrangian in the heavy quark limit. Beyond this symmetry limit, HQET is developed by expanding the QCD lagrangian in power of $1/m_Q$, in which heavy quark symmetry breaking terms are studied order by order. The QCD lagrangian for the heavy quark is as:

$$\mathcal{L}_Q = \bar{Q}(i\gamma_\mu D^\mu - m_Q)Q \quad (1)$$

Where $D^\mu \equiv \partial^\mu - igA^\mu$. As the interaction of this heavy quark with light degree of freedom is through the exchange of soft gluons, which is much smaller than the m_Q , so heavy quark momentum p_Q is

$$p_Q^\mu = m_Q v^\mu + k^\mu \quad (2)$$

In this $m_Q v^\mu$ is the kinetic momentum which comes from the mesons's motion and k^μ represents the residual momentum which is of the order of Λ_{QCD} . In the $m_Q \rightarrow \infty$ limit, redefining new heavy quark field $h_v(x)$, such that it is related to the original field $Q(x)$ by

$$\frac{1 + \not{v}}{2} Q(x) = e^{-im_Q v \cdot x} h_v(x) \quad (3)$$

Field $h_v(x)$ satisfies

$$\frac{1 + \not{v}}{2} h_v = h_v, \not{v} \gamma_\mu h_v(x) = k^\mu h_v(x) \quad (4)$$

From these relations equation 1 can be reduced to

$$\mathcal{L}_Q \rightarrow \mathcal{L}_{Q,eff} = \bar{h}_v (i v \cdot D) h_v \quad (5)$$

This lagrangian is invariant under both flavor and spin symmetry, since it is independent of heavy quark mass m_Q and the $\vec{\gamma}$ matrix respectively. Applying finite heavy quark mass corrections, HQET lagrangian to order of $1/m_Q$ is

$$\mathcal{L} = \bar{h}_v (i v \cdot D) h_v + \bar{h}_v \frac{(i D_\perp)^2}{2m_Q} h_v + \bar{h}_v \frac{g \sigma_{\mu\nu} G^{\mu\nu}}{4m_Q} h_v + \mathcal{O}(\frac{1}{m_Q^2}) \quad (6)$$

Where, $D_\perp^\mu = D^\mu - v^\mu v \cdot D$ is orthogonal to heavy quark velocity v , and $G^{\mu\nu} = T_a G_a^{\mu\nu} = \frac{2}{g_s} [D^\mu, D^\nu]$ is the gluon field strength tensor. In the limit $m_Q \rightarrow \infty$, only first term $\bar{h}(i v \cdot D) h$ survives. This symmetry is broken by the higher order terms in this lagrangian involving terms of factor $1/m_Q$. The second term D_\perp^2 is arising from the off shell residual momentum of the heavy quark in the non relativistic model and it represents the heavy quark kinetic energy $\frac{p_Q^2}{2m_Q}$ [25]. This term breaks the flavor symmetry because of the explicit dependence on m_Q , but does not break the spin symmetry of the HQET. The third term in the above equation i.e. $g \sigma_{\mu\nu} G^{\mu\nu}$ represents the magnetic moment interaction coupling of the heavy quark spin to the gluon field. This term breaks both the flavor and spin symmetry. This term is also known as magnetic chromo-magnetic term. From equation 6, it is seen that heavy quark symmetry is the symmetry of lowest order of $\mathcal{L}_{Q,eff}$, therefore the predictions from this heavy quark symmetry are model independent. We will not consider higher order corrections as we are interested only upto first order corrections in $(1/m_Q)$ expansion. Heavy quark symmetry is used to establish relations between hadron masses. At m_Q order, all hadrons containing same Q are degenerate, i.e. have the same mass m_Q [26]. At the order of unity, the $\frac{1}{m_Q^0}$

terms of HQET Hamiltonian (H_0) obtained from the first term of lagrangian defined in equation 1 and from the terms involving light quarks and gluons give contribution to hadron masses as

$$\frac{1}{2}\langle H^{(Q)} | H_0 | H^{(Q)} \rangle \equiv \bar{\Lambda} \quad (7)$$

At the $1/m_Q$ order, there is an extra addition to the hadron masses resulting from the contribution coming from the expectation value of the $1/m_Q$ correction to the Hamiltonian i.e. $H_1 = -L_1$. Matrix element of two terms in equation 3, define two more non-perturbative parameters λ_1 and λ_2 defined as:

$$2\lambda_1 = -\langle H^{(Q)} | \bar{h}D_\perp^2 h | H^{(Q)} \rangle \quad (8)$$

and

$$16(S_Q.S_l)\lambda_2(m_Q) = \alpha(\mu)\langle H^{(Q)} | \bar{h}g\sigma_{\mu\nu}G^{\mu\nu}h | H^{(Q)} \rangle \quad (9)$$

From these two non-perturbative parameters, λ_1 is independent of m_Q and other parameter λ_2 depends on m_Q through the logarithmic m_Q dependence of $\alpha(\mu)$ as:

$$\alpha(\mu) = \left[\frac{\alpha_s(m_Q)}{\alpha_s(\mu)} \right]^{9/(33-2N_q)} \quad (10)$$

Sine $\gamma^0 h = h$, the matrix element $\bar{h}\sigma_{\mu\nu}G^{\mu\nu}h$ reduces to the $\bar{h}\sigma.Bh$, where B is the chromomagnetic field. The operator $\bar{h}\sigma h$ represents the heavy quark spin and the matrix component of B in the heavy hadron represents the spin of the light degrees of freedom. So the contribution to mass from the third term i.e. the chromomagnetic operator contribution is proportional to $S_Q.S_l$. Thus the non-perturbative parameter of this term i.e. λ_2 transforms like $S_Q.S_l$ under the spin symmetry.

2.1 Masses

The mass of the heavy-light hadron to the first order of $1/m_Q$ in terms of these non-perturbative parameters can be represented as :

$$M_X = m_Q + \bar{\Lambda} - \frac{\lambda_1}{2m_Q} + 4(S_Q.S_l)\frac{\lambda_2}{2m_Q} \quad (11)$$

In this equation, $d_H = -4(S_Q.S_l)$ is the Clebsch factor. The two parameters λ_1 and λ_2 have the same value for all the hadrons with same spin-flavor multiplet. The values of these parameters is of the order of Λ_{QCD}^2 . Since value of kinetic energy of the heavy quark is positive, the value of the parameter λ_1 should be negative. $\bar{\Lambda}$ is the HQET parameter whose value is same for all the particles in a spin-flavor multiplet. The value of $\bar{\Lambda}_H$ for H field mesons is denoted by $\bar{\Lambda}_S$, for S fields by $\bar{\Lambda}_S$ and for T field by $\bar{\Lambda}_T$ and so on. $\bar{\Lambda}$ does not depend on the light quark flavor if there is $SU(3)$ symmetry, but for the breaking of this symmetry $\bar{\Lambda}$ is different for strange and non-strange heavy -light mesons and is denoted by $\bar{\Lambda}_s$ and $\bar{\Lambda}_{u,d}$ respectively.

Sometimes mass is also written as:

$$M_X = m_Q + \bar{\Lambda} + \frac{\Delta m^2}{2m_Q} + O\left(\frac{1}{m_Q^2}\right) \quad (12)$$

Where Δm^2 is related to the total spin J of the meson and is given by:

$$\Delta m^2 = -\lambda_1 + 2[J(J+1) - \frac{3}{2}]\lambda_2 \quad (13)$$

In these equations X is the hadron in any state, either in ground state(H) or in excited state(S) or (T), m_Q is the mass of the heavy quark either c (charm) or b (bottom) making the hadron and J is the total spin of the meson and λ_1, λ_2 are the two non perturbative QCD parameters. $\bar{\Lambda}$ and λ_1 can not be simply estimated by mass measurements on dimensional grounds. The parameter $\bar{\Lambda}$ gives the energy of the light degrees of freedom in the limit $m_Q \rightarrow \infty$.

Neglecting $SU(3)$ flavor symmetry breaking, mass relations for the lowest lying pseudoscalar and vector mesons of $J^P = 0^-$ and 1^- respectively i.e. for H fields, D and D^* for $Q = c$ and B and B^* for $Q = b$ mesons are parameterized as:

$$m_H = m_Q + \bar{\Lambda}^H - \frac{\lambda_1^H}{2m_Q} - 3\frac{\lambda_2^H}{2m_Q} + O(\frac{1}{m_Q^2}) \quad (14)$$

$$m_{H^*} = m_Q + \bar{\Lambda}^H - \frac{\lambda_1^H}{2m_Q} + \frac{\lambda_2^H}{2m_Q} + O(\frac{1}{m_Q^2}) \quad (15)$$

These equations for first orbitally excited state ($l = 1$) changes as shown below. Mass relations for spin $S_q = \frac{1}{2}$ i.e. for S field mesons are

$$m_S = m_Q + \bar{\Lambda}^S - \frac{\lambda_1^S}{2m_Q} - 3\frac{\lambda_2^S}{2m_Q} + O(\frac{1}{m_Q^2}) \quad (16)$$

$$m_{S^*} = m_Q + \bar{\Lambda}^S - \frac{\lambda_1^S}{2m_Q} + \frac{\lambda_2^S}{2m_Q} + O(\frac{1}{m_Q^2}) \quad (17)$$

Similarly for doublet ($1^+, 2^+$) belonging to spin ($S_q = \frac{3}{2}$) i.e. for T fields, these relation changes as:

$$m_T = m_Q + \bar{\Lambda}^T - \frac{\lambda_1^T}{2m_Q} - 5\frac{\lambda_2^T}{2m_Q} + O(\frac{1}{m_Q^2}) \quad (18)$$

$$m_{T^*} = m_Q + \bar{\Lambda}^T - \frac{\lambda_1^T}{2m_Q} + 3\frac{\lambda_2^T}{2m_Q} + O(\frac{1}{m_Q^2}) \quad (19)$$

These formulas for the difference of spin averaged masses can be written as:

$$\bar{m}_S^{(Q)} - \bar{m}_H^{(Q)} = \bar{\Lambda}^S - \bar{\Lambda}^H - \frac{\lambda_1^S}{2m_Q} + \frac{\lambda_1^H}{2m_Q} \quad (20)$$

$$\bar{m}_T^{(Q)} - \bar{m}_H^{(Q)} = \bar{\Lambda}^T - \bar{\Lambda}^H - \frac{\lambda_1^T}{2m_Q} + \frac{\lambda_1^H}{2m_Q} \quad (21)$$

Where $\bar{m}_H^{(Q)} = (3m_{H^*}^{(Q)} + m_H^{(Q)})/4$, $\bar{m}_S^{(Q)} = (3m_{S^*}^{(Q)} + m_S^{(Q)})/4$ and $\bar{m}_T^{(Q)} = (5m_{T^*}^{(Q)} + 3m_T^{(Q)})/8$. Different parameters $\bar{\Lambda}$, λ_1 and λ_2 appear for different fields. When $SU(3)$ symmetry is breaking, these parameters are again different for light quarks u, d and s . Using the above relations and the heavy quark symmetry, some more relations can be written as [27]:

$$\frac{m_{H^*}^b - m_H^b}{m_{H^*}^c - m_H^c} = \frac{m_{S^*}^b - m_S^b}{m_{S^*}^c - m_S^c} = \frac{m_{T^*}^b - m_T^b}{m_{T^*}^c - m_T^c} = \frac{m_c}{m_b} \quad (22)$$

Masses of the heavy hadrons are used to calculate their properties like strong decays, radiative decays, magnetic moments etc. So masses can be justified if we know some of the above properties accurately. In our work, to justify the masses, we study their strong decay widths.

2.2 Strong Decays

Strong interactions are very important for the study of heavy hadrons containing one heavy and one light quark in the non-perturbative regime. Heavy meson decay to light pseudo-scalar meson depends on the initial mass of the heavy hadron and on the quantum numbers of the decaying resonance. Strong decays are calculated by approaching heavy meson doublet in effective fields and imposing the heavy quark spin and flavor symmetry on it [11]. Strong decay width formulae for $l = 0, 1$ states decaying to various states are as follow: $(0^-, 1^-) \rightarrow (0^-, 1^-) + M$

$$\Gamma(1^- \rightarrow 1^-) = C_M \frac{g_{HH}^2 M_f p_M^3}{3\pi f_\pi^2 M_i} \quad (23)$$

$$\Gamma(1^- \rightarrow 0^-) = C_M \frac{g_{HH}^2 M_f p_M^3}{6\pi f_\pi^2 M_i} \quad (24)$$

$$\Gamma(0^- \rightarrow 1^-) = C_M \frac{g_{HH}^2 M_f p_M^3}{2\pi f_\pi^2 M_i} \quad (25)$$

$$(0^+, 1^+) \rightarrow (0^-, 1^-) + M$$

$$\Gamma(1^+ \rightarrow 1^-) = C_M \frac{g_{SH}^2 M_f (p_M^2 + m_M^2) p_M}{2\pi f_\pi^2 M_i} \quad (26)$$

$$\Gamma(0^+ \rightarrow 0^-) = C_M \frac{g_{SH}^2 M_f (p_M^2 + m_M^2) p_M}{2\pi f_\pi^2 M_i} \quad (27)$$

$$(0^-, 1^-) \rightarrow (0^+, 1^+) + M$$

$$\Gamma(1^- \rightarrow 1^+) = C_M \frac{g_{SH}^2 M_f (p_M^2 + m_M^2) p_M}{2\pi f_\pi^2 M_i} \quad (28)$$

$$\Gamma(0^- \rightarrow 0^+) = C_M \frac{g_{SH}^2 M_f (p_M^2 + m_M^2) p_M}{2\pi f_\pi^2 M_i} \quad (29)$$

$$(1^+, 2^+) \rightarrow (0^-, 1^-) + M$$

$$\Gamma(2^+ \rightarrow 1^-) = C_M \frac{2g_{TH}^2 M_f p_M^5}{5\pi f_\pi^2 \Lambda^2 M_i} \quad (30)$$

$$\Gamma(2^+ \rightarrow 0^-) = C_M \frac{4g_{TH}^2 M_f p_M^5}{15\pi f_\pi^2 \Lambda^2 M_i} \quad (31)$$

$$\Gamma(1^+ \rightarrow 1^-) = C_M \frac{2g_{TH}^2 M_f p_M^5}{3\pi f_\pi^2 \Lambda^2 M_i} \quad (32)$$

$$(0^-, 1^-) \rightarrow (1^+, 2^+) + M$$

$$\Gamma(1^- \rightarrow 2^+) = C_M \frac{2g_{TH}^2 M_f p_M^5}{3\pi f_\pi^2 \Lambda^2 M_i} \quad (33)$$

$$\Gamma(1^- \rightarrow 1^+) = C_M \frac{2g_{TH}^2 M_f p_M^5}{3\pi f_\pi^2 \Lambda^2 M_i} \quad (34)$$

$$\Gamma(0^- \rightarrow 2^+) = C_M \frac{4g_{TH}^2 M_f p_M^5}{3\pi f_\pi^2 \Lambda^2 M_i} \quad (35)$$

$$(0^-, 1^-) \rightarrow (1^-, 2^-) + M$$

$$\Gamma(1^- \rightarrow 2^-) = C_M \frac{10g_{HX}^2 M_f (p_M^2 + m_M^2) p_M^3}{9\pi f_\pi^2 \Lambda^2 M_i} \quad (36)$$

$$\Gamma(1^- \rightarrow 1^-) = C_M \frac{2g_{HX}^2 M_f (p_M^2 + m_M^2) p_M^3}{9\pi f_\pi^2 \Lambda^2 M_i} \quad (37)$$

$$\Gamma(0^- \rightarrow 1^-) = C_M \frac{4g_{HX}^2 M_f (p_M^2 + m_M^2) p_M^3}{3\pi f_\pi^2 \Lambda^2 M_i} \quad (38)$$

$$(0^+, 1^+) \rightarrow (1^-, 2^-) + M$$

$$\Gamma(1^+ \rightarrow 2^-) = C_M \frac{2g_{XS}^2 M_f p_M^5}{3\pi f_\pi^2 \Lambda^2 M_i} \quad (39)$$

$$\Gamma(1^+ \rightarrow 1^-) = C_M \frac{2g_{XS}^2 M_f p_M^5}{3\pi f_\pi^2 \Lambda^2 M_i} \quad (40)$$

$$\Gamma(0^+ \rightarrow 2^-) = C_M \frac{4g_{XS}^2 M_f p_M^5}{3\pi f_\pi^2 \Lambda^2 M_i} \quad (41)$$

$$(1^+, 2^+) \rightarrow (0^-, 1^-) + M$$

$$\Gamma(2^+ \rightarrow 2^-) = C_M \frac{17g_{TX}^2 M_f (p_M^2 + m_M^2) p_M^5}{45\pi f_\pi^2 \Lambda^2 M_i} \quad (42)$$

$$\Gamma(2^+ \rightarrow 1^-) = C_M \frac{g_{TX}^2 M_f (p_M^2 + m_M^2) p_M^5}{15\pi f_\pi^2 \Lambda^2 M_i} \quad (43)$$

$$\Gamma(1^+ \rightarrow 2^-) = C_M \frac{g_{TX}^2 M_f (p_M^2 + m_M^2) p_M^5}{9\pi f_\pi^2 \Lambda^2 M_i} \quad (44)$$

$$\Gamma(1^+ \rightarrow 1^-) = C_M \frac{g_{TX}^2 M_f (p_M^2 + m_M^2) p_M^5}{3\pi f_\pi^2 \Lambda^2 M_i} \quad (45)$$

$$(0^-, 1^-) \rightarrow (2^-, 3^-) + M$$

$$\Gamma(1^- \rightarrow 3^-) = C_M \frac{16g_{HY}^2 M_f p_M^7}{45\pi f_\pi^2 \Lambda^4 M_i} \quad (46)$$

$$\Gamma(1^- \rightarrow 2^-) = C_M \frac{4g_{HY}^2 M_f p_M^7}{9\pi f_\pi^2 \Lambda^4 M_i} \quad (47)$$

$$\Gamma(0^- \rightarrow 3^-) = C_M \frac{4g_{HY}^2 M_f p_M^7}{5\pi f_\pi^2 \Lambda^4 M_i} \quad (48)$$

$$(0^+, 1^+) \rightarrow (2^-, 3^-) + M$$

$$\Gamma(1^+ \rightarrow 3^-) = C_M \frac{28g_{SY}^2 M_f (p_M^2 + m_M^2) p_M^5}{45\pi f_\pi^2 \Lambda^4 M_i} \quad (49)$$

$$\Gamma(1^+ \rightarrow 2^-) = C_M \frac{8g_{SY}^2 M_f (p_M^2 + m_M^2) p_M^5}{45\pi f_\pi^2 \Lambda^4 M_i} \quad (50)$$

$$\Gamma(0^+ \rightarrow 2^-) = C_M \frac{4g_{SY}^2 M_f (p_M^2 + m_M^2) p_M^5}{5\pi f_\pi^2 \Lambda^4 M_i} \quad (51)$$

$$(1^+, 2^+) \rightarrow (2^-, 3^-) + M$$

$$\Gamma(2^+ \rightarrow 3^-) = C_M \frac{28g_{TY}^2 M_f p_M^5}{75\pi f_\pi^2 \Lambda^2 M_i} \quad (52)$$

$$\Gamma(2^+ \rightarrow 2^-) = C_M \frac{7g_{TY}^2 M_f p_M^5}{75\pi f_\pi^2 \Lambda^2 M_i} \quad (53)$$

$$\Gamma(1^+ \rightarrow 3^-) = C_M \frac{14g_{TY}^2 M_f p_M^5}{135\pi f_\pi^2 \Lambda^2 M_i} \quad (54)$$

$$\Gamma(1^+ \rightarrow 2^-) = C_M \frac{49g_{TY}^2 M_f p_M^5}{135\pi f_\pi^2 \Lambda^2 M_i} \quad (55)$$

In the above expressions of decay widths, M_i, M_f stands for initial and final meson mass. All hadronic coupling constants are dependent on the radial quantum number, for $n=1$ they are notated as g_{HH}, g_{SH} etc, and for coupling between $n=2$ and $n=1$ they will be replaced by $\tilde{g}_{HH}^2, \tilde{g}_{SH}^2$ etc and similarly for the coupling between initial and final states both belonging to $n=2$, they are again replaced by $\tilde{g}_{HH}^2, \tilde{g}_{SH}^2$ etc. These notations can be made clear from Figure 1. Λ is the chiral symmetry breaking scale = 1GeV , p_M and m_M is the final momentum and mass of the emitted light pseudo-scalar meson. The coefficient $C_{\pi^\pm}, C_{K^\pm}, C_{K^0}, C_{\bar{K}^0} = 1$, $C_{\pi^0} = \frac{1}{2}$ and $C_\eta = \frac{2}{3}$ or $\frac{1}{6}$. Different values of C_η corresponds to the initial state being $c\bar{u}, c\bar{d}$ or $c\bar{s}$ respectively.

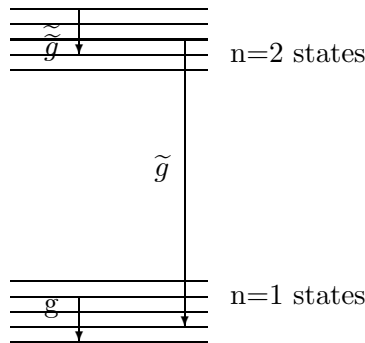


Figure 1: Figure showing the notation for various hadronic couplings

3 Calculations

Calculations of this work are subdivided into two parts one in which we estimate the masses of the bottom mesons and the other in which the calculated masses are used to predict their strong decay widths. Decay widths in terms of the hadronic coupling constants are predicted by constraining the hadronic coupling constants to lie in the range 0-1.

Mass relations given in equations 14-22 are used for $n=2$, charm and bottom mesons for J^P ($0^-, 1^-$), ($0^+, 1^+$) and ($1^+, 2^+$) doublets. These relations are used to fit the values of the parameters present in these equations ($\bar{\Lambda}^H, \lambda_1^H, \lambda_2^H, \bar{\Lambda}^S, \lambda_1^S, \lambda_2^S, \bar{\Lambda}^T, \lambda_1^T$ and λ_2^T). Due to the lack of experimental or theoretical information on these values for radial quantum number $n=2$, we choose to fit the differences $\bar{\Lambda}^H - \bar{\Lambda}^S, \bar{\Lambda}^H - \bar{\Lambda}^T, \lambda_1^H - \lambda_1^S$ and $\lambda_1^H - \lambda_1^T$, rather than the separate parameters. As λ_2 parameter gives the mass difference between the same doublet, so it can be calculated easily once the masses are known. For this fitting, charm meson masses for $n=2$ and the heavy quark symmetry is used. Here heavy quark symmetry implies that the values of these parameters for charm mesons are same for the corresponding bottom mesons. Radial charm meson masses for $J^P(0^-, 1^-)$ doublet are experimentally available as [28] $\tilde{D} = 2550$ MeV and $\tilde{D}^* = 2600$ MeV. Masses for two doublets of $l=1$ (p-wave) are not known experimentally, but are theoretically predicted by some models [29,30]. Using the experimental masses of $J^P(0^-, 1^-)$ doublet and taking range of P wave masses as estimated by the models, we fitted the values of the differences of the parameters as $\bar{\Lambda}^H - \bar{\Lambda}^S = 0.29$ GeV, $\bar{\Lambda}^H - \bar{\Lambda}^T = 0.31$ GeV, $\lambda_1^H - \lambda_1^S = 0.28$ GeV² and $\lambda_1^H - \lambda_1^T = 0.30$ GeV² for the charm mesons as $\tilde{D}_0^* = 2940$ MeV, $\tilde{D}_1' = 3021$ MeV, $\tilde{D}_1 = 3031$ MeV and $\tilde{D}_2^* = 3032$ MeV. SU(3) breaking gives $\bar{\Lambda}_s^H - \bar{\Lambda}_s^S = 0.24$ GeV, $\bar{\Lambda}_s^H - \bar{\Lambda}_s^T = 0.28$ GeV, $\lambda_{1s}^H - \lambda_{1s}^S = 0.27$ GeV² and $\lambda_{1s}^H - \lambda_{1s}^T = 0.30$ GeV² for strange charm masses $\tilde{D}_s = 2688$ MeV and $\tilde{D}_s^* = 2731$ MeV $\tilde{D}_{s0}^* = 3050$ MeV, $\tilde{D}_{s1}' = 3094$ MeV, $\tilde{D}_{s1} = 3110$ MeV and $\tilde{D}_{s2}^* = 3150$ MeV. This fitting is done for both strange and non-strange radially excited mesons with charm and bottom quark masses as $m_c = 1.18$ GeV $m_b = 4.39$ GeV. This provides the masses for the bottom mesons which are tabulated in 2 and 5 column of Table 2.

$J^P(n^{2s+1}L_J)$	Masses of $n=2$ Bottom Mesons(MeV)					
	Non-Strange			Strange		
	Calculated	[29]	[30]	Calculated	[29]	[30]
$0^-(2^1S_0)$	5940.64	5890	5886	6022.30	5976	5985
$1^-(2^3S_1)$	5954.04	5906	5920	6033.80	5992	6019
$0^+(2^3P_0)$	6260.84	6221	6163	6301.10	6318	6264
$1^+(2P_1)$	6282.61	6281	6194	6313.00	6345	6296
$1^+(2P_1)$	6301.10	6209	6175	6340.61	6321	6278
$2^+(2^3P_2)$	6301.14	6260	6188	6341.14	6359	6292

Table 2: Theoretically predicted bottom meson masses. Column 2 and 5 represents the non-strange and strange bottom meson masses calculated in our framework, which are compared with masses predicted by other theoretical approaches.

These calculated bottom mesons are then used to find calculate the λ_2 , which comes out to be $\lambda_2^H = 29.413$ GeV², $\lambda_2^S = 47.76$ GeV² and $\lambda_2^T = 0.087$ GeV². Similarly for the strange mesons, this parameter for different fields is calculated as $\lambda_{2s}^H = 25.24$ GeV², $\lambda_{2s}^S = 26.12$ GeV² and $\lambda_{2s}^T = 1.16$

GeV^2 .

Our calculated masses, are now justified by calculating the strong decay width for J^P ($0^-, 1^-$), ($0^+, 1^+$) and ($1^+, 2^+$) doublets of bottom meson. Initial masses of these states are taken from our calculated values mentioned in Table 2, and the masses of rest of the particles are taken from Ref. [5, 29]. Decay channels along with their decay widths are presented in Table 3 and in Table 4, for calculated bottom mesons without and with strangeness. Column 3 of these Tables shows the possible decay channels, whose widths are shown in column 4. In Column 5, we calculated the total width after using the available values of hadronic couplings constants in literature [31, 32].

State	$J^P(2^{s+1}L_J)$	Decay Channels	Width (MeV)	Total Width (MeV)
B (5940.6)	$0^-(2^1S_0)$	$B^{*0}\pi^0$ $B^{*+}\pi^-$ $B^{*0}\eta^0$ $B_s^*K^0$ $B_0^{*0}\pi^0$ $B_0^{*+}\pi^-$ $B_2^{*+}\pi^-$	$778.71\tilde{g}_{HH}^2$ $1579.23\tilde{g}_{HH}^2$ $26.3268\tilde{g}_{HH}^2$ $34.6502\tilde{g}_{HH}^2$ $45.5883\tilde{g}_{SH}^2$ $89.9833\tilde{g}_{SH}^2$ $0.698535\tilde{g}_{HH}^2$	189.93
B^* (5954.0)	$1^-(2^3S_1)$	$B^0\pi^0$ $B^+\pi^-$ $B^0\eta^0$ B_sK^0 $B^{*0}\pi^0$ $B^{*+}\pi^-$ $B^{*0}\eta^0$ $B_s^*K^0$ $B_1^{'0}\pi^0$ $B_1^{'+}\pi^-$ $B_1^0\pi^0$ $B_2^{*0}\pi^0$	$337.037\tilde{g}_{HH}^2$ $671.146\tilde{g}_{HH}^2$ $23.6149\tilde{g}_{HH}^2$ $73.5802\tilde{g}_{HH}^2$ $552.55\tilde{g}_{HH}^2$ $1099.55\tilde{g}_{HH}^2$ $23.2967\tilde{g}_{HH}^2$ $42.7305\tilde{g}_{HH}^2$ $32.0787\tilde{g}_{SH}^2$ $62.6883\tilde{g}_{SH}^2$ $2.4879\tilde{g}_{TH}^2$ $1.67433\tilde{g}_{TH}^2$	222.44
B_0^* (6260.84)	$0^+(2^3P_0)$	$B^0\pi^0$ $B^+\pi^-$ $B^0\eta$ B_sK^0 $\tilde{B}^0\pi^0$ $\tilde{B}^+\pi^-$ $B_2^0\pi^0$ $B_2^+\pi^-$ $B_2^{'0}\pi^0$ $B_2^{'+}\pi^-$	$2918.45\tilde{g}_{SH}^2$ $5833.81\tilde{g}_{SH}^2$ $854.935\tilde{g}_{SH}^2$ $4019.36\tilde{g}_{SH}^2$ $228.942\tilde{g}_{SH}^2$ $455.986\tilde{g}_{SH}^2$ $2.00822\tilde{g}_{XS}^2$ $3.63042\tilde{g}_{XS}^2$ $0.381716\tilde{g}_{YS}^2$ $0.722983\tilde{g}_{YS}^2$	$136.26+684.92\tilde{g}_{SH}^2+5.63\tilde{g}_{XS}^2+1.10\tilde{g}_{YS}^2$
B_1 (6282.61)	$1^+(2P_1)$	$\tilde{B}^{*0}\pi^0$ $\tilde{B}^{*+}\pi^-$ $B^{*0}\pi^0$ $B^{*+}\pi^-$ $B^{*0}\eta^0$	$41.1618\tilde{g}_{SH}^2$ $80.8277\tilde{g}_{SH}^2$ $2742.63\tilde{g}_{SH}^2$ $5482.63\tilde{g}_{SH}^2$ $795.318\tilde{g}_{SH}^2$	

		$B_s^* K^0$ $B_1^{*0} \pi^0$ $B_1^{*+} \pi^-$ $B_2^0 \pi^0$ $B_2^+ \pi_-$ $B_2'^0 \pi^0$ $B_2'^+ \pi_-$ $B_3^{*0} \pi^0$ $B_3^{*0} \pi^0$	$4324.27 \tilde{g}_{SH}^2$ $2.76024 \tilde{g}_{XS}^2$ $5.16546 \tilde{g}_{XS}^2$ $1.98803 \tilde{g}_{XS}^2$ $3.68453 \tilde{g}_{XS}^2$ $0.158338 \tilde{g}_{SY}^2$ $0.241994 \tilde{g}_{SY}^2$ $0.444255 \tilde{g}_{SY}^2$ $0.84698 \tilde{g}_{SY}^2$	$133.44 + 121.98 \tilde{g}_{SH}^2 + 13.59 \tilde{g}_{XS}^2 + 1.69 \tilde{g}_{YS}^2$
$B_1'(6301.10)$	$1^+(2P_1')$	$\tilde{B}^{*0} \pi^0$ $\tilde{B}^{*+} \pi^-$ $B^{*0} \pi^0$ $B^{*+} \pi^-$ $B_1^{*0} \pi^0$ $B_1^{*+} \pi^-$ $B_2^0 \pi^0$ $B_2^+ \pi_-$ $B_2'^0 \pi^0$ $B_2'^+ \pi_-$ $B_3^{*0} \pi^0$ $B_3^{*+} \pi^-$	$2.00253 \tilde{g}_{TH}^2$ $3.71221 \tilde{g}_{TH}^2$ $2989.85 \tilde{g}_{TH}^2$ $5959.36 \tilde{g}_{TH}^2$ $0.160504 \tilde{g}_{TX}^2$ $0.30404 \tilde{g}_{TX}^2$ $0.0367408 \tilde{g}_{TX}^2$ $0.0691139 \tilde{g}_{TX}^2$ $5.45427 \tilde{g}_{TY}^2$ $10.4917 \tilde{g}_{TY}^2$ $1.33718 \tilde{g}_{TY}^2$ $2.56558 \tilde{g}_{TY}^2$	$289.95 + 5.71 \tilde{g}_{TH}^2 + 0.57 \tilde{g}_{TX}^2 + 19.84 \tilde{g}_{TY}^2$
$B_2^*(6301.14)$	$2^+(2^3P_2)$	$\tilde{B}^0 \pi^0$ $\tilde{B}^+ \pi^-$ $\tilde{B}^{*0} \pi^0$ $\tilde{B}^{*+} \pi^-$ $B^{*0} \pi^0$ $B^{*+} \pi^-$ $B^{*0} \eta^0$ $B_s^* K^0$ $B^0 \pi^0$ $B^+ \pi^-$ $B^0 \eta$ $B_s K^0$ $B_1^{*0} \pi^0$ $B_1^{*+} \pi^-$ $B_2^0 \pi^0$ $B_2^+ \pi_-$ $B_2'^0 \pi^0$ $B_2'^+ \pi_-$ $B_3^{*0} \pi^0$ $B_3^{*0} \pi^0$	$22.995 \tilde{g}_{TH}^2$ $45.1337 \tilde{g}_{TH}^2$ $1.20289 \tilde{g}_{TH}^2$ $2.22995 \tilde{g}_{TH}^2$ $1794.25 \tilde{g}_{TH}^2$ $3576.29 \tilde{g}_{TH}^2$ $242.376 \tilde{g}_{TH}^2$ $1344.32 \tilde{g}_{TH}^2$ $3.02909 \tilde{g}_{TH}^2$ $3.05293 \tilde{g}_{TH}^2$ $1.17829 \tilde{g}_{TH}^2$ $896.214 \tilde{g}_{TH}^2$ $0.0321397 \tilde{g}_{TX}^2$ $0.0608831 \tilde{g}_{TX}^2$ $0.125081 \tilde{g}_{TX}^2$ $0.235298 \tilde{g}_{TX}^2$ $1.40358 \tilde{g}_{TY}^2$ $2.69992 \tilde{g}_{TY}^2$ $4.81759 \tilde{g}_{TY}^2$ $9.24343 \tilde{g}_{TY}^2$	$254.68 + 71.56 \tilde{g}_{TH}^2 + 0.45 \tilde{g}_{TX}^2 + 18.16 \tilde{g}_{TY}^2$

Table 3: Decay widths of calculated masses of non-strange bottom masses

State	$J^P(^{2s+1}L_J)$	Decay Channels	Width(MeV)	Total Width (MeV)
$B_s(6022.5)$	$0^-(2^1S_0)$	$B^{*0}K^0$ $B^{*+}K^-$ $B_s^*\pi^0$ $B_s^*\eta$ $B_{s0}^*\pi^0$ $B_{s2}^*\pi^0$	$807.889\tilde{g}_{HH}^2$ $827.816\tilde{g}_{HH}^2$ $749.546\tilde{g}_{HH}^2$ $84.8866\tilde{g}_{HH}^2$ $35.6177\tilde{g}_{HS}^2$ $0.312315\tilde{g}_{HS}^2$	194.32
$B_s^*(6033.8)$	$1^-(2^3S_1)$	B^0K^0 B^+K^- $B_s\pi^0$ $B_s\eta$ $B^{*0}K^0$ $B^{*+}K^-$ $B_s^*\pi^0$ $B_s^*\eta$ $B'_{s1}\pi^0$ $B_{s1}\pi^0$ $B_{s2}^*\pi^0$	$410.868\tilde{g}_{HH}^2$ $418.465\tilde{g}_{HH}^2$ $328.07\tilde{g}_{HH}^2$ $86.1936\tilde{g}_{HH}^2$ $591.59\tilde{g}_{HH}^2$ $603.842\tilde{g}_{HH}^2$ $527.747\tilde{g}_{HH}^2$ $74.789\tilde{g}_{HH}^2$ $22.1298\tilde{g}_{HS}^2$ $0.503379\tilde{g}_{HS}^2$ $0.291981\tilde{g}_{HS}^2$	239.46
$B_{s0}^*(6301.1)$	$0^+(2^3P_0)$	$\tilde{B}_s\pi^0$ B^0K^0 B^+K^- $B_s\pi^0$ $B_s\eta$ $B_{s2}\pi^0$ $B'_{s2}\pi^0$	$185.906\tilde{g}_{SH}^2$ $5935.95\tilde{g}_{SH}^2$ $5947.22\tilde{g}_{SH}^2$ $2577.88\tilde{g}_{SH}^2$ $2958.0\tilde{g}_{SH}^2$ $0.18314\tilde{g}_{SX}^2$ $0.0256384\tilde{g}_{SY}^2$	$174.19+185.90\tilde{g}_{SH}^2+0.18\tilde{g}_{XS}^2+0.02\tilde{g}_{YS}^2$
$B_{s1}(6313.0)$	$1^+(2P_1)$	$\tilde{B}_s^*\pi^0$ $B^{*0}K^0$ $B^{*+}K^-$ $B_s^*\pi^0$ $B_s^*\eta$ $B_{s1}^*\pi^0$ $B_{s2}\pi^0$ $B'_{s2}\pi^0$ $B_{s3}^*\pi^0$	$186.491\tilde{g}_{SH}^2$ $5400.68\tilde{g}_{SH}^2$ $5411.62\tilde{g}_{SH}^2$ $2319.87\tilde{g}_{SH}^2$ $2609.36\tilde{g}_{SH}^2$ $0.194189\tilde{g}_{SX}^2$ $0.0845814\tilde{g}_{SX}^2$ $0.0101309\tilde{g}_{SY}^2$ $0.0242715\tilde{g}_{SY}^2$	$157.41+186.49\tilde{g}_{SH}^2+0.27\tilde{g}_{XS}^2+0.03\tilde{g}_{YS}^2$
$B'_{s1}(6340.61)$	$1^+(2P'_1)$	$\tilde{B}_s^*\pi^0$ $B_s^*\pi^0$ $B_{s1}^*\pi^0$ $B_{s2}\pi^0$	$34.7237\tilde{g}_{TH}^2$ $2353.35\tilde{g}_{TH}^2$ $0.0154948\tilde{g}_{TX}^2$ $0.00266403\tilde{g}_{TX}^2$	$76.24+34.72\tilde{g}_{TH}^2+0.01\tilde{g}_{TX}^2+0.24\tilde{g}_{TY}^2$

		$B'_{s2}\pi^0$	$1.08377\tilde{g}_{TY}^2$	
		$B^*_{s3}\pi^0$	$0.2445\tilde{g}_{TY}^2$	
$B^*_{s2}(6341.14)$	$2^+(2^3P_2)$	$\tilde{B}^*_s\pi^0$	$20.9877\tilde{g}_{TH}^2$	$342.24+37.34\tilde{g}_{TH}^2+0.01\tilde{g}_{TX}^2+1.17\tilde{g}_{TY}^2$
		$\tilde{B}_s\pi^0$	$16.36\tilde{g}_{TH}^2$	
		B^0K^0	$1949.31\tilde{g}_{TH}^2$	
		B^+K^-	$1971.47\tilde{g}_{TH}^2$	
		$B_s\pi^0$	$0.552817\tilde{g}_{TH}^2$	
		$B_s\eta$	$0.934714\tilde{g}_{TH}^2$	
		$B^{*0}K^0$	$2261.54\tilde{g}_{TH}^2$	
		$B^{*+}K^-$	$2289.93\tilde{g}_{TH}^2$	
		$B^*_s\pi^0$	$1415.76\tilde{g}_{TH}^2$	
		$B^*_s\eta$	$673.684\tilde{g}_{TH}^2$	
		$B^*_{s1}\pi^0$	$0.00317786\tilde{g}_{TX}^2$	
		$B_{s2}\pi^0$	$0.00932343\tilde{g}_{TX}^2$	
		$B_{s2}\pi^0$	$0.282934\tilde{g}_{TY}^2$	
		$B^*_{s3}\pi^0$	$0.894524\tilde{g}_{TY}^2$	

Table 4: Decay widths of calculated masses of strange bottom masses

4 Conclusion

Last year LHC [1] predicted bottom states, which are assigned the J^P as 1^+ and 2^+ in 1P bottom sector. Experimental information for the radial excited states 2S, 2P,.. is still missing. In this paper, we try to shed some light on the masses and decays of these radial excited 2S and 2P states, by analyzing them in the heavy quark effective theory. At the $\frac{1}{m_Q}$ order, the bottom meson masses are related to some parameters like $\bar{\Lambda}$, λ_1 and λ_2 . Using the heavy quark symmetry and the available charm meson masses, we fitted the $\bar{\Lambda}^H - \bar{\Lambda}^S$, $\bar{\Lambda}^H - \bar{\Lambda}^T$, $\lambda_1^H - \lambda_1^S$ and $\lambda_1^H - \lambda_1^T$ to attain the masses for the bottom states. For some best fitted values of these differences, our predicted masses are comparable with other theoretical models. Masses calculated in our frame work are about 50 MeV large than the masses obtained from Ref. [29] and about 126 MeV large than the values predicted by Ref. [30]. This difference between the masses, can be reduced by getting a clear information on the unknown parameters. We assume that values of the differences of these non-perturbative parameters for n=2 are comparable with their differences for n=1 states, which is based on that, the difference of these parameters is independent of the radial number, so that, they can be used in future to predict the masses of heavy-light mesons for n=3 quantum number.

Along with the mass prediction, we studied the OZI allowed two body strong decay to light pseudo-scalar mesons (π, η, K). Column 4 of Table 3 and Table 4 shows the contribution of various possible decay channels to the total decay width in terms of the various hadronic coupling constants. As the experimental information about these hadronic couplings is very limited, so we used the available theoretical values like $\tilde{g}_{HH}^2=0.28$ [32], $\tilde{g}_{SH}^2=0.1$ [31] and $\tilde{g}_{TH}^2=0.18$ [32], and calculated the total decay width of these radially excited states in column 5 of these Tables. $\Gamma(0^-)$ and $\Gamma(1^-)$ without strangeness comes out to be 189 MeV and 222 MeV respectively, and for their strange partners decay widths comes out to be 194MeV and 239 MeV respectively.

As it can be seen from the column 5 of these Tables, that the contribution to the total decay width from the decays to X and Y fields is very small, so even if we vary the values of these couplings $\tilde{g}_{SX}^2, \tilde{g}_{SY}^2, \tilde{g}_{TX}^2$ and \tilde{g}_{TY}^2 from 0 to 1, the total decay width would not effect the result much.

We are still left with two more higher hadronic couplings \tilde{g}_{SH}^2 and \tilde{g}_{TH}^2 . As there is no experimental information for the decays of these bottom states, the values of these couplings from 0-1 would effect the total decay width to a greater extent. To give some insight to these higher order couplings, we studied the decay widths for higher charm meson states. As the states $D_s(3040)$ and $D(3000)$ and $D^*(3000)$ are expected to fit in 2P charm spectra, so using their experimental decay widths, we can constrain their couplings to be approximately $\tilde{g}_{SH}^2=0.1$ and $\tilde{g}_{TH}^2=0.3$ for the hadronic coupling $\tilde{g}_{SH}^2=0.14$ and $\tilde{g}_{TH}^2=0.12$ unlike before.

So using $\tilde{g}_{SH}^2=0.1, \tilde{g}_{TH}^2=0.3, \tilde{g}_{SX}^2=0.14$ and $\tilde{g}_{TY}^2=0.12$, we calculated the total decay width of the n=2 S and P wave bottom states, which are shown in Table 5. Column 3 of Table 5 shows that the total decay width deviation for strange states is small due to the hadronic coupling g_{SX}, g_{SY}, g_{TX} and g_{TY} .

The decay width of T-wave states $B_1'(6228), B_2^*(6213), B_{s1}'(6296)$ and $B_{s2}^*(6295)$ comes out to be $\Gamma(B_1') = 148 MeV, \Gamma(B_2^*) = 82 MeV, \Gamma(B_{s1}') = 30 MeV$ and $\Gamma(B_{s2}^*) = 90 MeV$ respectively when only decays to light pseudo-scalar mesons are considered in 3P_0 model [23]. These values are comparable with $B_1'(6228), B_2^*(6213), B_{s1}'(6296)$ and $B_{s2}^*(6295)$ states decay widths $\Gamma(B_1') = 139 MeV, \Gamma(B_2^*) = 128 MeV, \Gamma(B_{s1}') = 37 MeV$ and $\Gamma(B_{s2}^*) = 156 MeV$ respectively as calculated in our framework. These decay widths for non-strange and strange bottom mesons are a kind of motivation for the theorists and experimentalists to look for them with their proper J^P states to have a clear idea.

$J^P(n^{2s+1}L_J)$	Predicted decay Width of n=2 Bottom Mesons(MeV)	
	Non-Strange	Strange
$0^-(2^1S_0)$	189.95	194.36
$1^-(2^3S_1)$	223.27	238.67
$0^+(2^3P_0)$	277.29 ± 3.36	343.37 ± 0.10
$1^+(2P_1)$	270.41 ± 7.64	310.54 ± 0.15
$1^+(2P_1)$	139.58 ± 10.20	37.13 ± 0.12
$2^+(2^3P_2)$	128.935 ± 9.30	156.06 ± 0.59

Table 5: Theoretically predicted decay width of n=2 bottom meson masses. Column 2 and 3 represents the non-strange and strange bottom meson decay widths calculated by us.

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